THE ASTROPHYSICAL JOURNAL, **221**:274–283, 1978 April 1

SUPERDENSE NEUTRON MATTER

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Received 1977 August 22; accepted 1977 October 10

ABSTRACT

We present a relativistic theory of high-density matter which takes into account the short-range interaction due to the exchange of spin-2 mesons. An equation of state is derived and used to compute neutron star properties. The prediction of our theory for the values of maximum mass and moment of inertia for a stable neutron star are 1.75 $M_\odot$ and 1.68×10^{45} g cm^2, in very good agreement with the presently known observational bounds. The corresponding radius is found to be 10.7 km.

We find that the inclusion of the spin-2 interaction reduces the disagreement between the relativistic and nonrelativistic theories in their predictions of masses and moments of inertia.

Subject headings: dense matter — equation of state — pulsars — stars: neutron

I. INTRODUCTION

Although masses of stable neutron stars have been computed by many authors in recent years, there still remains disagreement concerning the upper limit (Canuto 1977). This is due to a lack of clear theoretical understanding of (i) the interactions among very densely packed neutrons and (ii) the methods to be employed to study dense matter (Canuto 1974, 1975). Results for the gravitational mass as predicted by relevant computations are presented in Figure 1, which is taken from Canuto (1977). On the right-hand side of Figure 1 we include observational results due to Joss and Rappaport (1976) and Avni (1976); the ranges of masses given by these authors are, respectively, 1.4–1.84 $M_\odot$ and 1–2.3 $M_\odot$. Curves A–G are the results of nonrelativistic computations, and they yield maximum masses less than 1.84 $M_\odot$. Curve L is the result of computations by Pandharipande and Smith (1975), who included pion tensor interaction in a nonrelativistic way. For the high-density region that prevails in neutron stars a nonrelativistic description is of uncertain reliability. Furthermore, a recent analysis by Brown (1976) has shown that much of the repulsion inherent in Pandharipande and Smith's work is actually canceled if higher order terms are included.

Curve O is from the work of Bowers, Gleeson, and Pedigo (1975*a*, *b*), who adopted an effective relativistic Lagrangian.

Curve N is the result of a relativistic calculation by Walecka (1974), who considered scalar and vector interactions in a Hartree scheme. A general review of these models can be found in Canuto (1974, 1975, 1977). In all these computations, nonrelativistic as well as relativistic, it is assumed that the high-density regime is dominated by the exchange of the vector mesons, which produce repulsion among neutrons. However, when the nucleon-nucleon separation is very short ($\leq$0.4 fermi [fm]), attractive forces due to the exchange of spin-2 mesons must come into play. The objective of this paper is to present the results of a relativistic many-body theory of high-density neutrons in which, for the first time, forces deriving from the exchange of spin-2 mesons have been included, in addition to the traditional scalar (spin 0) and vector (spin 1) interactions.

We find that the inclusion of the spin-2 interaction brings important and so far unpredictable new features into the behavior of high-density matter. It provides the dominant force at high densities, where it makes the equation of state rather soft. Beyond a critical density, the pressure decreases monotonically with increasing density, approaching a negative asymptotic value. The formalism used in the treatment of the spin-2 interaction is presented in § II. The many-body calculation has been confined to zero temperature, since neutron stars are essentially a zero-temperature system. The computations of the equation of state and the astrophysical applications are presented in § III. We find that the present theory, by including the spin-2 interaction, is able to reduce the disagreement

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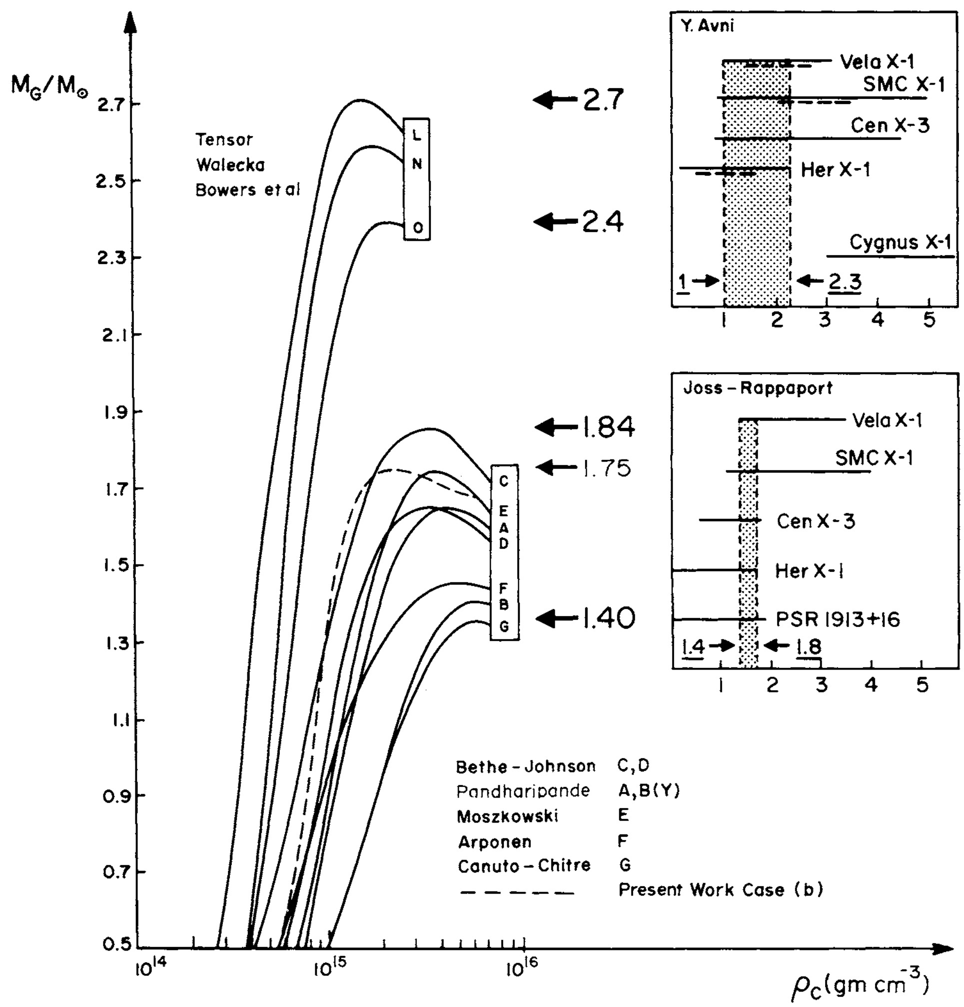

FIG. 1.—Neutron-star gravitational mass (in units of the solar mass) versus central density. For curves A–O, see Canuto (1977). The horizontal scales in the two insets refer to masses of the pulsars in units of the solar mass.

in the values of the neutron-star maximum mass as predicted by the existing theoretical models. The prediction of our theory for the values of the maximum mass and moment of inertia are 1.75 $M_\odot$ and 1.68×10^{45} g cm^2, in satisfactory agreement with the observational bounds. The corresponding radius is 10.7 km.

II. THE MODEL

The full nonlinear theory of an interacting spin-2 field has been worked out by the present authors (Canuto, Datta, and Kalman 1977) and will be published in a separate paper elsewhere. Here we shall summarize the main results and present the relevant equations. The equations of motion for the field operators ψ, σ, A_μ, and $g_{\mu\nu}$ that determine the behavior of a Fermi gas coupled to a scalar (σ), a vector (A_μ), and a spin-2 ($g_{\mu\nu}$) field, are derived to be ($\hbar = 1 = c$)

$$\{e^\mu{}_a\gamma^a i^{-1}(\nabla_\mu - ig_v A_\mu) + m_N - g_\sigma\sigma\}\psi = 0\,, \tag{1}$$

$$(-\partial^2 + m_\sigma^2)\sigma = g_\sigma\bar{\psi}\psi\,, \tag{2}$$

$$m_v^2(\sqrt{-g})A_\mu h^{\mu\nu} + \partial_\mu[(\sqrt{-g})F_{\lambda\rho}h^{\lambda\nu}h^{\rho\mu}] = g_v(\sqrt{-g})\bar{\psi}e^\nu{}_a\gamma^a\psi\,, \tag{3}$$

$$R_{\mu\nu} + m_f^2[(\sqrt{-g})g_{\mu\nu} - \eta_{\mu\nu}] = \frac{16\pi f^2}{m_N^2}(t_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}t_{\alpha\beta}h^{\alpha\beta})\,. \tag{4}$$

In writing the above, we have used a formal analogy with gravitation. The mass term in equation (4) has been constructed in such a way that when sources are absent, the equation reduces to the familiar Pauli-Fierz equation for the free massive spin-2 field. The source term $t_{\mu\nu}$ is the stress tensor corresponding to all the fields except the free spin-2 field, and is given by

$$t_{\mu\nu} = \bar{\psi} d_{\mu a}\gamma^a i^{-1}(\nabla_\nu - i g_v A_\nu)\psi + \partial_\mu\sigma\partial_\nu\sigma - \tfrac{1}{2}g_{\mu\nu}(\partial_\lambda\sigma h^{\lambda\rho}\partial_\rho\sigma + m_\sigma^2\sigma^2) + m_v^2 A_\mu A_\nu - \tfrac{1}{2}m_v^2 g_{\mu\nu}A_\alpha h^{\alpha\beta}A_\beta + F_{\mu\alpha}F_{\beta\nu}h^{\alpha\beta} - \tfrac{1}{4}g_{\mu\nu}F_{\alpha\beta}h^{\alpha\lambda}h^{\beta\rho}F_{\lambda\rho} \,. \tag{5}$$

The quantities $e^\mu{}_a$ and $d_{\mu a}$ are *Vierbein* fields given by ($\mu, a = 0, 1, 2, 3$)

$$e^{\mu a}e^{\nu b}\eta_{ab} = e^{\mu a}e^\nu{}_b = h^{\mu\nu} = g_{\mu\nu}{}^{-1} \tag{6}$$

$$d_\mu{}^a d_\nu{}^b \eta_{ab} = d_\mu{}^a d_{\nu a} = g_{\mu\nu} \tag{7}$$

where $\eta_{ab} = \text{diag.}\,(-1, 1, 1, 1)$.

The aim of our microscopic theory is to evaluate the conserved total stress tensor of the system, which is given by

$$T^\mu{}_\nu = \theta^\mu{}_\nu + (-g)^{1/2}h^{\mu\beta}t_{\beta\nu} \,, \tag{8}$$

where $\theta_{\mu\nu}$ is the stress tensor of the free spin-2 field. We evaluate $T_{\mu\nu}$ by resorting to the Hartree approximation. This consists in assuming that the fermions "see" an average effective cloud of mesons. Such an approximation can be justified when nucleons are very dense so that source terms in the meson field equations are large and quantum fluctuations about the expectation values of the meson field are small. Thus, we shall take the meson Hartree fields to be c-numbers, and denote them as

$$\sigma \rightarrow \sigma_0 \,,$$

$$A_\mu \rightarrow \delta_\mu{}^0 A_0 \,,$$

$$\bar{g}^{\mu\nu} \equiv (\sqrt{-g})h^{\mu\nu} = \begin{pmatrix} -(1+\chi) & 0 & 0 & 0 \\ 0 & 1+\lambda & 0 & 0 \\ 0 & 0 & 1+\lambda & 0 \\ 0 & 0 & 0 & 1+\lambda \end{pmatrix} , \tag{9}$$

where σ_0, A_0, χ, and λ are spacetime independent. We shall take the source terms appearing in equations (2)–(4) to correspond to average quantities. The averages are performed by expanding the fermion field in terms of creation and annihilation operators and choosing the prescription of normal ordering for these operators. From the conservation of the baryonic current,

$$\partial_\mu[(\sqrt{-g})\bar{\psi}e^\mu{}_a\gamma^a\psi] = 0 \,,$$

we shall identify the average value of $(\sqrt{-g})\bar{\psi}e_a{}^0\gamma^a\psi$ with the baryonic number density n. The latter, which is related to the Fermi momentum k_F via the relation $n = \gamma k_F{}^3/6\pi^2$ (γ = spin degeneracy factor), will be taken as the independent parameter of the theory.

With the mean field approximation (9), the meson field equations (2)–(4) take the following form

$$\sigma_0 = \frac{\gamma g_\sigma}{4\pi^2 xy}(m_N - g_\sigma\sigma_0)J_1 \,, \tag{10}$$

$$A_0 = -\frac{\gamma g_v}{6\pi^2 m_v{}^2}\frac{k_F{}^3}{x^2} \,, \tag{11}$$

$$x^4 - x^4y^6 - \frac{\gamma f^2 xy}{\pi m_f{}^2 m_N{}^2}J_2 + \frac{2\gamma f^2(m_N - g_\sigma\sigma_0)^2}{\pi m_f{}^2 m_N{}^2}x^2y^2J_1 + \frac{8\pi f^2 m_\sigma{}^2 x^3y^3}{m_N{}^2 m_f{}^2}\sigma_0{}^2 - \frac{4\gamma^2 f^2 g_v{}^2 k_F{}^6}{9\pi^3 m_f{}^2 m_N{}^2 m_v{}^2} = 0 \,, \tag{12}$$

$$3x^3y^5 - 3xy - \frac{\gamma f^2}{\pi m_f{}^2 m_N{}^2}J_3 - \frac{6\gamma f^2(m_N - g_\sigma\sigma_0)^2}{\pi m_f{}^2 m_N{}^2}xyJ_1 = \frac{24\pi f^2 m_\sigma{}^2 x^2y^2\sigma_0{}^2}{m_f{}^2 m_N{}^2} \,, \tag{13}$$

where

$$x^2 = 1 + \chi\,, \qquad y^2 = 1 + \lambda\,,$$

$$J_1 = \frac{y}{2\pi x}\int_0^{k_F} d^3k(k^{*2} + m_N{}^{*2})^{-1/2}\,,$$

$$J_2 = \frac{x}{\pi y}\int_0^{k_F} d^3k(k^{*2} + m_N{}^{*2})^{1/2}\,,$$

$$J_3 = \frac{x}{\pi y}\int_0^{k_F} k^{*2}d^3k(k^{*2} + m_N{}^{*2})^{-1/2}\,,$$

$$(1 + \chi)k^{*2} = (1 + \lambda)k^2\,,$$

$$(1 + \chi)^{1/2}m_N{}^{*2} = (1 + \lambda)^{3/2}(m_N - g_\sigma\sigma_0)^2\,. \tag{14}$$

For a given value of k_F, equations (10)–(12) can be solved numerically to give the values of the meson Hartree fields σ_0, A_0, χ, and λ.

III. THE EQUATION OF STATE—ASTROPHYSICAL APPLICATIONS

The equation of state is obtained by determining the density dependence of both pressure and energy density. In order to achieve this, we must first determine self-consistently the meson Hartree fields. For purpose of actual calculations, we have identified the scalar, vector, and spin-2 mesons with σ (700 MeV), ω (784 MeV), and f^0 (1260 MeV) (Particle Data Group 1974). Correspondingly, the coupling constants for scalar and vector mesons have been taken to be: $g_\sigma{}^2/4\pi = 13.9$ and $g_\omega{}^2/4\pi = 10.0$ (Pilkuhn *et al.* 1973). The f^0-meson coupling constant is not known accurately. Three experimental values are cited by Pilkuhn *et al.* (1973): $f^2 = 2.91$, 6.55, and 7.44.

Assuming our system to be isotropic, the pressure P and the total energy density ϵ follow from the diagonal components of $T_{\mu\nu}$. Evaluation of (8) yields the following expressions for P and ϵ:

$$P = \frac{3m_f{}^2m_N{}^2\lambda}{32\pi f^2}[\chi + (1 + \lambda)(1 + \chi)] + \frac{\gamma}{48\pi^2}\frac{y}{x}J_3 - \frac{m_\sigma{}^2}{2}(1 + \chi)^{1/2}(1 + \lambda)^{3/2}\sigma_0{}^2 + \frac{\gamma^2 g_v{}^2 k_F{}^6}{72\pi^4 m_v{}^2(1 + \chi)}\,, \tag{15}$$

$$\epsilon = -\frac{3m_f{}^2m_N{}^2\lambda}{32\pi f^2}[\chi + (1 + \lambda)(1 + \chi)] + \frac{\gamma}{16\pi^2}\frac{y}{x}J_2 + \frac{m_\sigma{}^2}{2}(1 + \chi)^{1/2}(1 + \lambda)^{3/2}\sigma_0{}^2 + \frac{\gamma^2 g_v{}^2 k_F{}^6}{72\pi^4 m_v{}^2(1 + \chi)}\,. \tag{16}$$

The pressure P and the energy per particle ϵ/n as functions of n are presented in Figures 2 and 3. The most conspicuous feature of this equation of state is that beyond a critical density (which depends on the values of the coupling

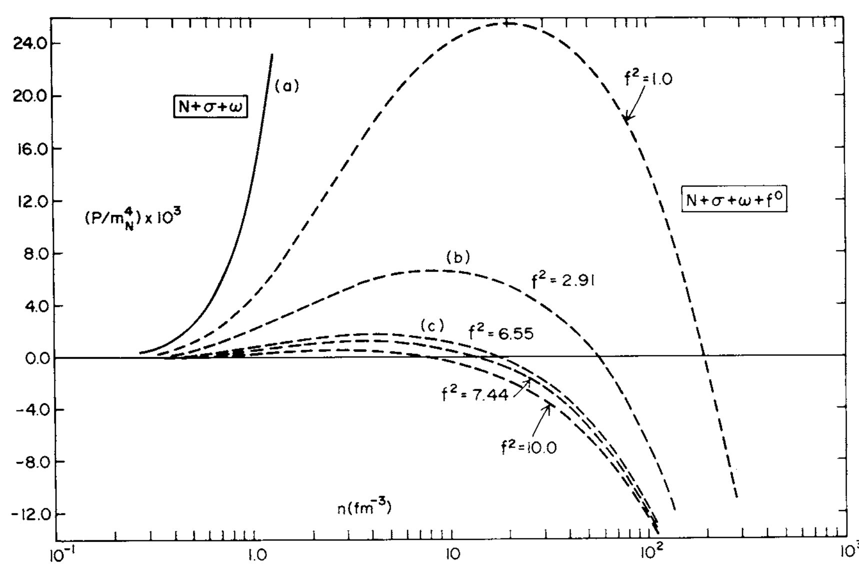

FIG. 2.—Pressure of neutron matter (in dimensionless units) versus neutron number density. The solid curve corresponds to the case where spin-2 mesons are absent.

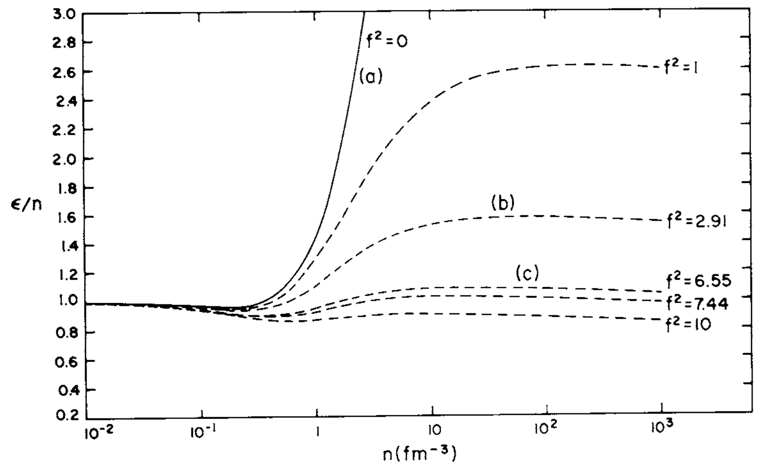

FIG. 3.—Energy per particle (in units of $m_N c^2$) versus neutron number density. The solid curve corresponds to the case where spin-2 mesons are absent.

constants and masses of the mesons), the pressure becomes a monotonically decreasing function of the baryon number density, reaching a negative asymptotic value.

To facilitate the comparison with the other equations of state, we present in Figure 4 all the equations of state used to compute the masses of Figure 1, as well as the results of the present work. Moreover, in Table 1 we present the numerical values of P, E/N, and $\rho = \epsilon/c^2$ versus n for two values of the spin-2 coupling constant. For densities lower than the last entry in Table 1 our results join smoothly with the values of P as given by Malone, Johnson, and Bethe (1975) (model V). With an equation of state so constructed, we have derived the values for the neutron star masses using standard methods. The results for the gravitational mass,

$$M_G = \int_0^R 4\pi r'^2 \rho(r')dr' ,$$

and the baryonic mass,

$$M_B = m_B \int_0^R \frac{4\pi r^2 n(r)dr}{(1 - 2Gm(r)/rc^2)^{1/2}}$$

TABLE 1

EQUATION OF STATE FOR NEUTRON MATTER

	$f^2 = 2.91$			$f^2 = 6.55$		
n (fm^{-3})	ρ (g cm^{-3})	E/N (GeV)	P (dyn cm^{-2})	ρ (g cm^{-3})	E/N (GeV)	P (dyn cm^{-2})
7.295	1.837 (16)	1.412	1.076 (36)	1.317 (16)	1.012	2.442 (35)
6.590	1.647 (16)	1.402	1.068 (36)	1.186 (16)	1.010	2.572 (35)
5.931	1.471 (16)	1.391	1.057 (36)	1.065 (16)	1.007	2.679 (35)
5.318	1.307 (16)	1.378	1.040 (36)	9.519 (15)	1.004	2.762 (35)
4.749	1.155 (16)	1.364	1.017 (36)	8.466 (15)	1.000	2.817 (35)
4.222	1.014 (16)	1.348	9.890 (35)	7.492 (15)	0.995	2.845 (35)
3.735	8.849 (15)	1.329	9.545 (35)	6.592 (15)	0.990	2.844 (35)
3.287	7.663 (15)	1.308	9.132 (35)	5.764 (15)	0.983	2.810 (35)
2.877	6.583 (15)	1.283	8.648 (35)	5.006 (15)	0.976	2.742 (35)
2.502	5.604 (15)	1.256	8.089 (35)	4.314 (15)	0.967	2.638 (35)
2.161	4.724 (15)	1.226	7.452 (35)	3.688 (15)	0.957	2.495 (35)
1.853	3.937 (15)	1.192	6.738 (35)	3.124 (15)	0.946	2.312 (35)
1.576	3.242 (15)	1.154	5.949 (35)	2.620 (15)	0.932	2.088 (35)
1.327	2.635 (15)	1.113	5.095 (35)	2.172 (15)	0.918	1.822 (35)
1.107	2.111 (15)	1.070	4.193 (35)	1.780 (15)	0.902	1.519 (35)
0.912	1.667 (15)	1.025	3.271 (35)	1.441 (15)	0.886	1.186 (35)
0.741	1.297 (15)	0.981	2.367 (35)	1.150 (15)	0.870	8.367 (34)

NOTE.—Numbers in parentheses indicate power of 10 by which each entry must be multiplied.

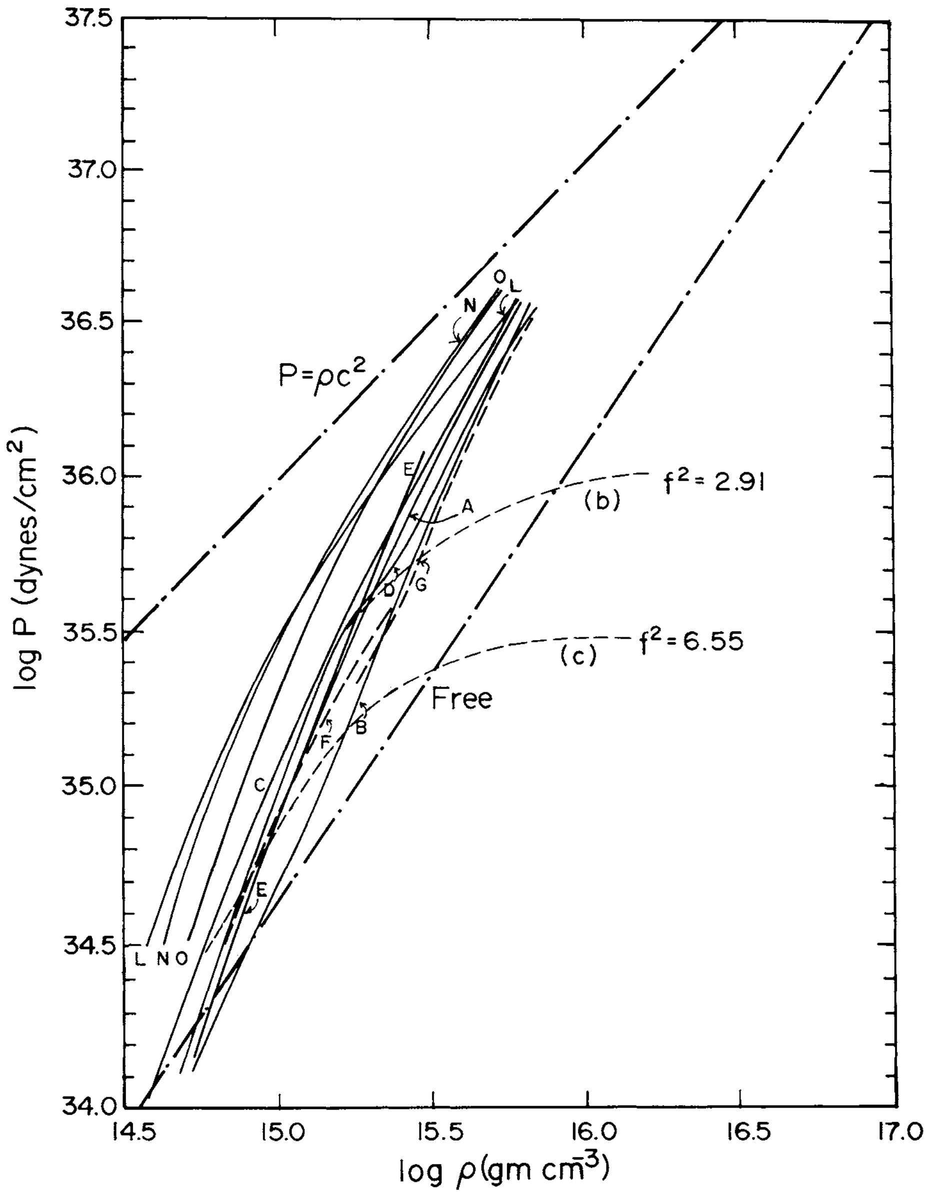

FIG. 4.—Pressure versus mass density. For curves A–O, see Canuto (1977). Curves (*b*) and (*c*) as from the present theory.

[R = radius of the star, m_B = neutron rest mass, $n(r)$ = number density], are shown in Figures 5 and 6; curve (*b*) of Figure 5 is then reproduced in Figure 1.

The maximum mass for a neutron star turns out to be 1.75 $M_\odot$. This value is significantly lower than the predictions of the previous relativistic calculations. In Table 2 we present the values of M_G, M_B, R, and moment of inertia I, as functions of central density for a specific value of the spin-2 coupling constant: $f^2 = 2.91$. Figure 7 shows the mass-radius relationship. Results for the moment of inertia are presented in Figure 8, and a comparison with the predictions of the other models is made in Figure 9. The two arrows in Figure 9 indicate the value of the moment of inertia needed if a neutron star is to be held responsible for the luminosity of the Crab Nebula as well as the kinetic energy of the expanding gas (Ruderman 1972). We see that our theory, case (*b*), can satisfy this requirement for masses in excess of 1 $M_\odot$.

IV. CONCLUSIONS

From Figure 1, we see that our result for the maximum mass for a stable neutron star (1.75 $M_\odot$) bunches together with the results of the models A–G. Thus incorporation of spin-2 interactions in a relativistic manner narrows the

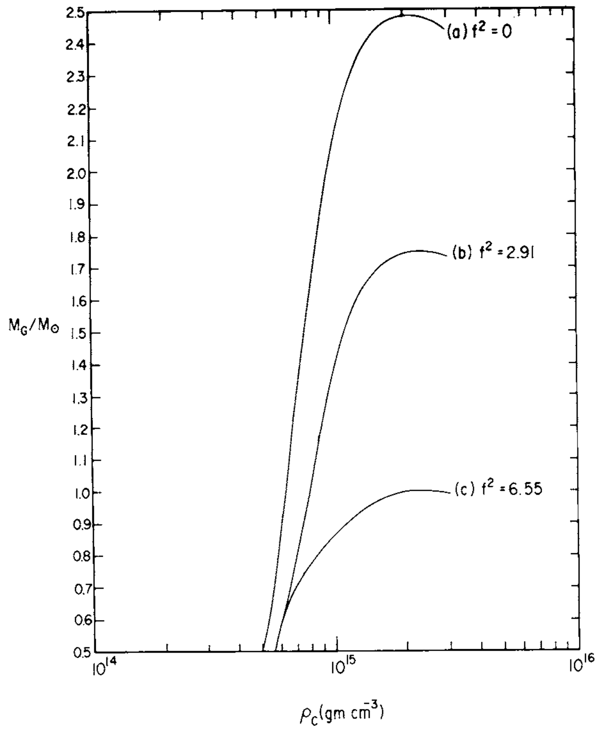

FIG. 5

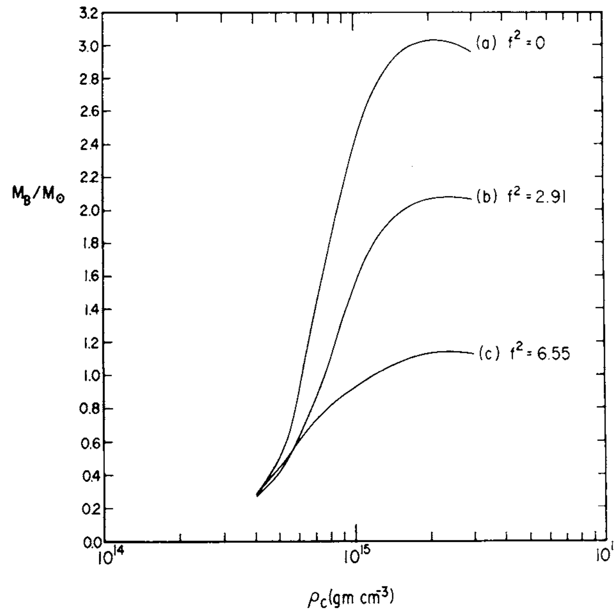

FIG. 6

FIG. 5.—Neutron-star gravitational mass, as predicted by the present theory, as a function of central density

FIG. 6.—Neutron-star baryonic mass, as predicted by the present theory, as a function of central density

disagreement between the sets of results A–G and L, N, O. We stress, however, that the apparent closeness of our result with those of the nonrelativistic models (A–G) cannot be interpreted as implying that the nonrelativistic theories provide an adequate description of high-density matter. The reason for this closeness is due to the competition of two important physical aspects of our theory, namely: (i) the inclusion of relativistic effects, and (ii) the treatment of the very short-range attractive *N-N* interaction, arising from the exchange of f^0 mesons. Thus, insofar as a fully relativistic theory is imperative to describe high-density matter, our theory clearly provides a more realistic and adequate description.

TABLE 2

NEUTRON-STAR PARAMETERS

ρ_c (g cm^{-3})	$M_G/M_\odot$	$M_B/M_\odot$	R (km)	I (g cm^2)
1.0 (16)......	1.65	1.96	10.17	1.33 (45)
8.0 (15)......	1.66	1.97	10.19	1.35 (45)
6.0 (15)......	1.68	1.99	10.24	1.39 (45)
4.0 (15)......	1.71	2.04	10.38	1.48 (45)
3.0 (15)......	1.74	2.07	10.56	1.56 (45)
2.4 (15)......	1.75	2.09	10.72	1.63 (45)
2.0 (15)......	1.74	2.07	10.88	1.67 (45)
1.8 (15)......	1.73	2.05	10.96	1.68 (45)
1.6 (15)......	1.71	2.01	11.06	1.67 (45)
1.4 (15)......	1.66	1.94	11.16	1.64 (45)
1.2 (15)......	1.57	1.80	11.29	1.53 (45)
1.0 (15)......	1.37	1.54	11.37	1.28 (45)
9.0 (14)......	1.23	1.35	11.40	1.09 (45)
7.0 (14)......	0.82	0.86	11.43	6.11 (44)
5.0 (14)......	0.41	0.41	12.17	2.40 (44)
3.0 (14)......	0.19	0.19	15.00	9.86 (43)

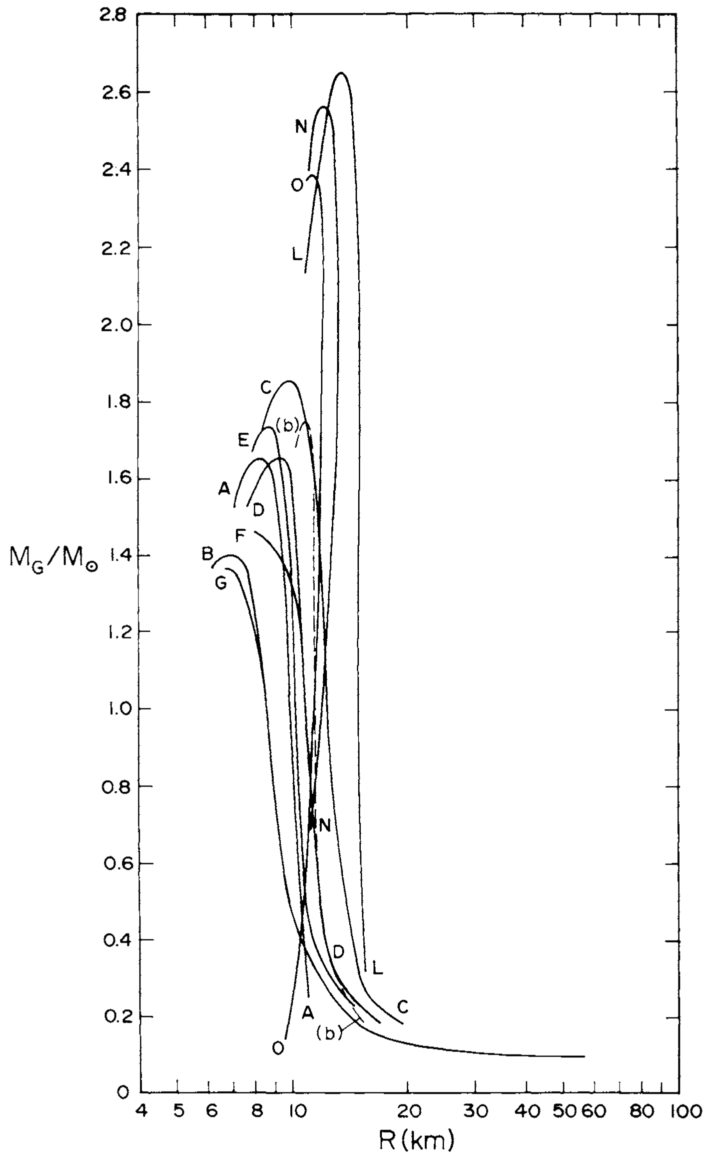

FIG. 7.—Neutron-star gravitational mass versus radius. For curves A–O, see Canuto (1977). Curve (*b*) as from the present theory.

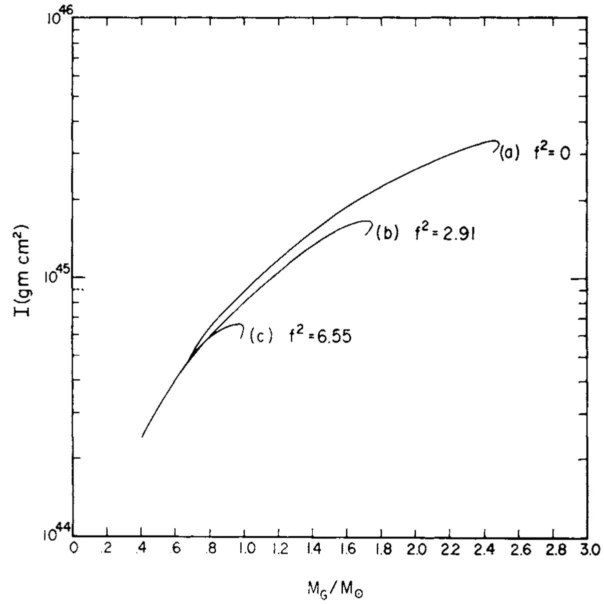

FIG. 8.—Moment of inertia versus gravitational mass, as predicted by the present theory

The present paper has not gone beyond the Hartree approximation in its treatment of the many-body theory of a neutron gas. However, we would like to emphasize that the consistent relativistic Hartree approximation is not equivalent to a lowest-order expansion in the coupling constant, and in that sense is a more exact theory.

Recently it has been suggested (Baym and Chin 1976) that neutron matter will undergo a phase transition to quark matter at high densities. If such is indeed the case, then the present model can bridge a critical gap in our understanding of dense matter in the region of density starting from nuclear matter density up to the densities where neutrons are expected to merge into a quark soup.

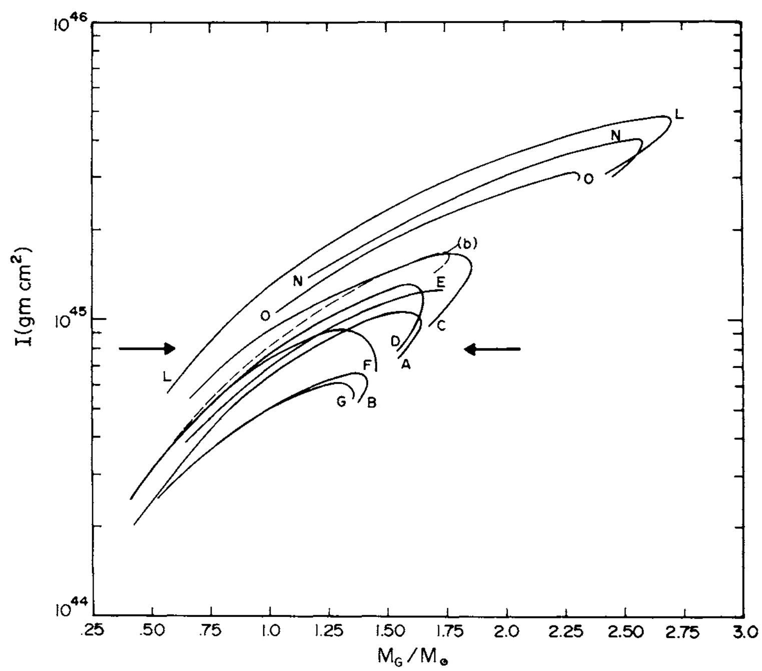

FIG. 9.—Moment of inertia versus gravitational mass. For curves A–O, see Canuto (1977). Curve (*b*) as from the present theory.

One of the authors (B. D.) wishes to thank Dr. R. Jastrow for hospitality at the Institute for Space Studies, NASA, New York.

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